



Meta-Amortized Variational Inference and Learning

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Probabilistic Inference

Probabilistic inference is a particular way of viewing the world:



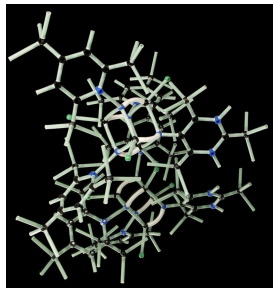
Typically the beliefs are “hidden” (unobserved), and we want to model them using latent variables.

Probabilistic Inference

Many machine learning applications can be cast as probabilistic inference queries:



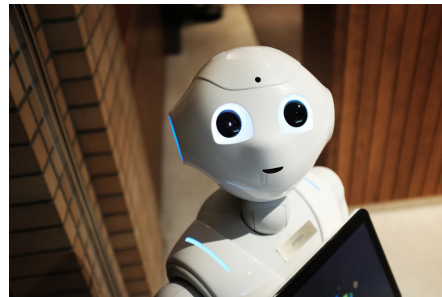
Medical diagnosis



Bioinformatics



Human cognition



Computer vision

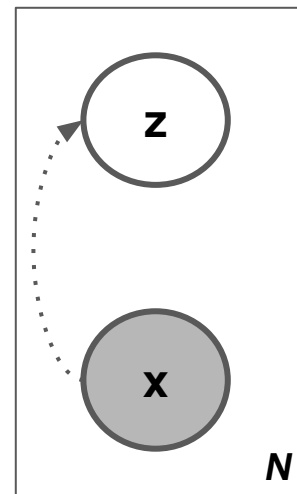
Medical Diagnosis Example

observed symptoms $\mathbf{x} \in \mathcal{X}$

identity of disease $\mathbf{z} \in \mathcal{Z}$



Goal: Infer identity of disease
given a set of observed symptoms
from a patient population.



Exact Inference



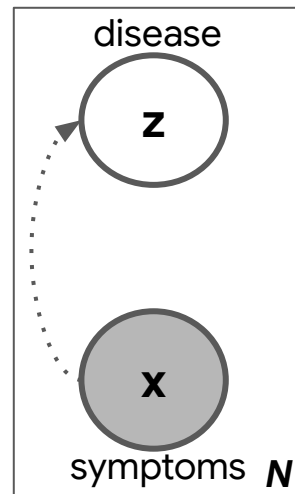
$$p(\mathbf{z}|\mathbf{x}) = p(\mathbf{x}, \mathbf{z}) / p(\mathbf{x})$$

\approx

family of tractable
distributions

$$q_{\psi} \in \mathcal{Q}$$

intractable integral $\int_{\mathbf{z}} p(\mathbf{x}, \mathbf{z}) d\mathbf{z}$



Marginal is intractable, we can't compute this even if we want to

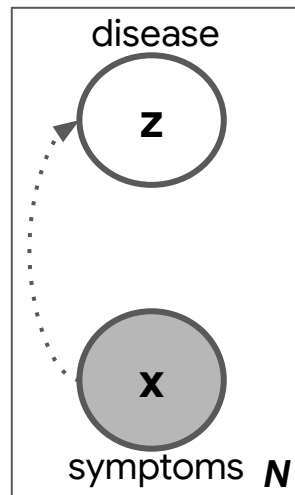
Approximate Variational Inference

dependence on \mathbf{x} : learn new q per data point

$$\mathbb{E}_{p_{\mathcal{D}}(\mathbf{x})} \left[\max_{\psi} \mathbb{E}_{q_{\psi}(\mathbf{z})} \log \frac{p(\mathbf{x}, \mathbf{z})}{q_{\psi}(\mathbf{z})} \right]$$



→ turned an intractable inference problem into an optimization problem

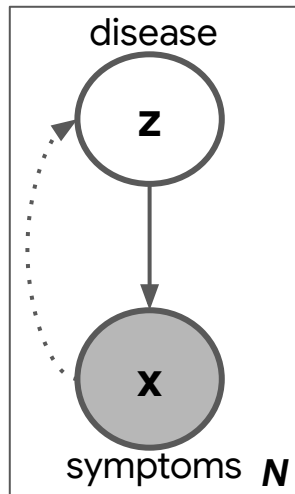


Amortized Variational Inference

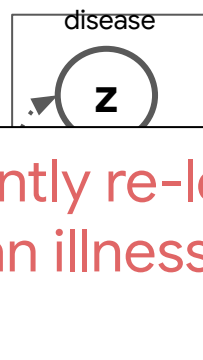
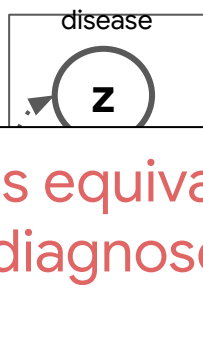
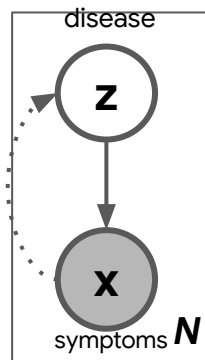
deterministic mapping predicts \mathbf{z} as a function of \mathbf{x}

$$\max_{\phi} \mathbb{E}_{p_{\mathcal{D}}(\mathbf{x})} \left[\mathbb{E}_{q_{\phi}(\mathbf{z}|\mathbf{x})} \log \frac{p_{\theta}(\mathbf{x}, \mathbf{z})}{q_{\phi}(\mathbf{z}|\mathbf{x})} \right]$$

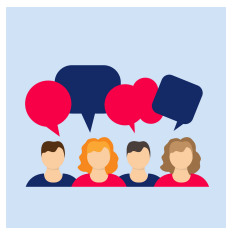
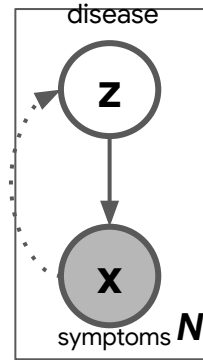
→ scalability: VAE formulation



Multiple Patient Populations



Doctor is equivalently re-learning
how to diagnose an illness :/



$p\mathcal{D}_1$



$p\mathcal{D}_2$



$p\mathcal{D}_3$



$p\mathcal{D}_4$

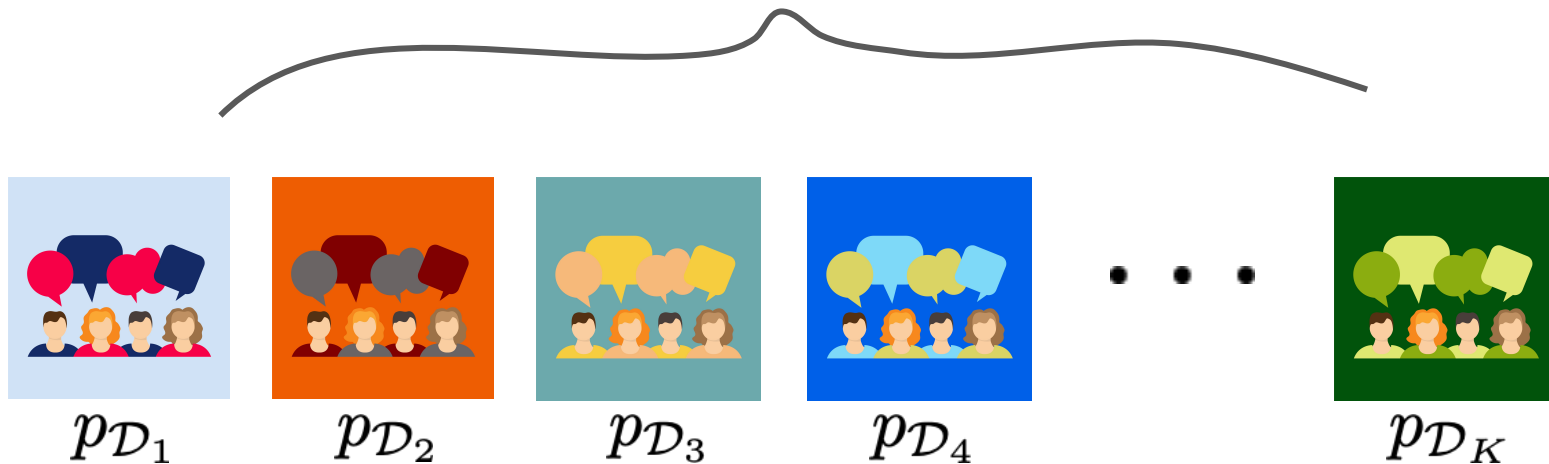
...




$p\mathcal{D}_K$

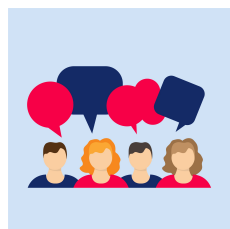
Multiple Patient Populations

Share statistical strength across different populations to infer latent representations that transfer to similar, but previously unseen populations (distributions)



(Naive) Meta-Amortized Variational Inference

$$\mathbb{E}_{p_{\mathcal{D}_i} \sim p_{\mathcal{M}}} \left[\max_{\phi} \mathbb{E}_{p_{\mathcal{D}_i}(\mathbf{x})} \left[\mathbb{E}_{q_{\phi}(\mathbf{z}|\mathbf{x})} \log \frac{p_{\theta_i}(\mathbf{x}, \mathbf{z})}{q_{\phi}(\mathbf{z}|\mathbf{x})} \right] \right]$$




$p_{\mathcal{D}_1}$



$p_{\mathcal{D}_2}$



$p_{\mathcal{D}_3}$

...



$p_{\mathcal{D}_K}$

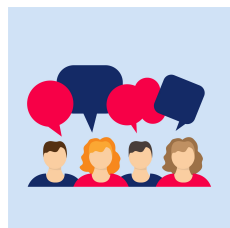
$\sim p_{\mathcal{M}}$

meta-distribution

Meta-Amortized Variational Inference

$$\max_{\phi} \mathbb{E}_{p_{\mathcal{D}_i} \sim p_{\mathcal{M}}} \left[\mathbb{E}_{p_{\mathcal{D}_i}(\mathbf{x})} \left[\mathbb{E}_{g_{\phi}(p_{\mathcal{D}_i}, \mathbf{x})} \log \frac{p_{\theta_i}(\mathbf{x}, \mathbf{z})}{g_{\phi}(p_{\mathcal{D}_i}, \mathbf{x})(\mathbf{z})} \right] \right]$$

shared meta-inference network



$p_{\mathcal{D}_1}$



$p_{\mathcal{D}_2}$



$p_{\mathcal{D}_3}$

...



$p_{\mathcal{D}_K}$

$\sim p_{\mathcal{M}}$

meta-distribution

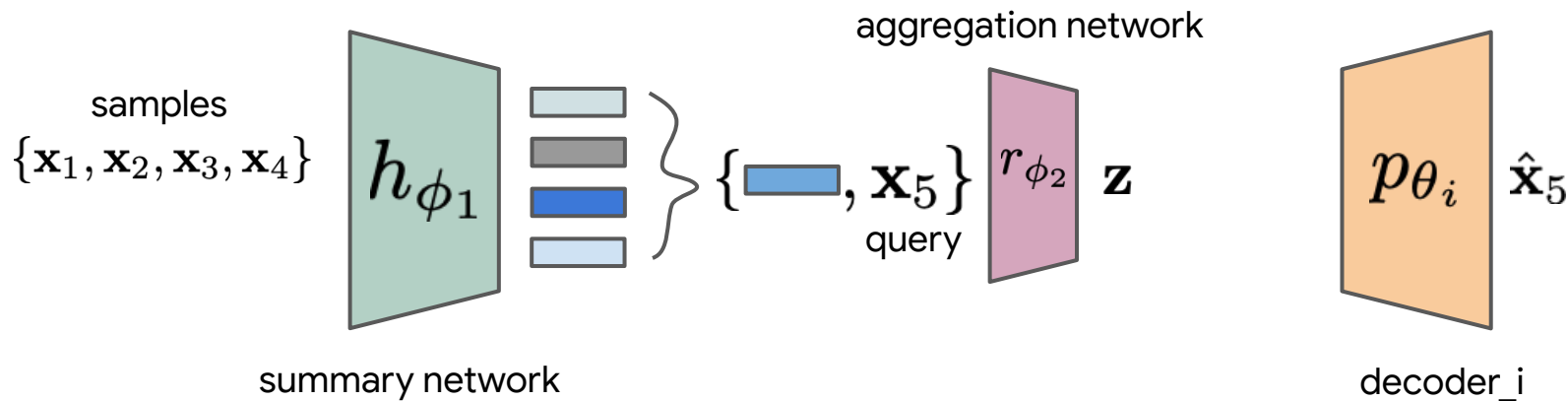
Meta-Inference Network

- Meta-inference model $g_\phi(p_{\mathcal{D}_i}, \mathbf{x})(\mathbf{z})$ takes in 2 inputs:
 - Marginal $p_{\mathcal{D}_i}$
 - Query point \mathbf{x}
- Mapping $g_\phi : \mathcal{M} \times \mathcal{X} \rightarrow \mathcal{Q}$
- Parameterize encoder with neural network
- Dataset \mathcal{D}_i : represent each marginal distribution as a set of samples

$$\mathcal{D}_i = \{\mathbf{x}_j \sim p_{\mathcal{D}_i}(\mathbf{x})\}_{j=1}^N$$

In Practice: MetaVAE

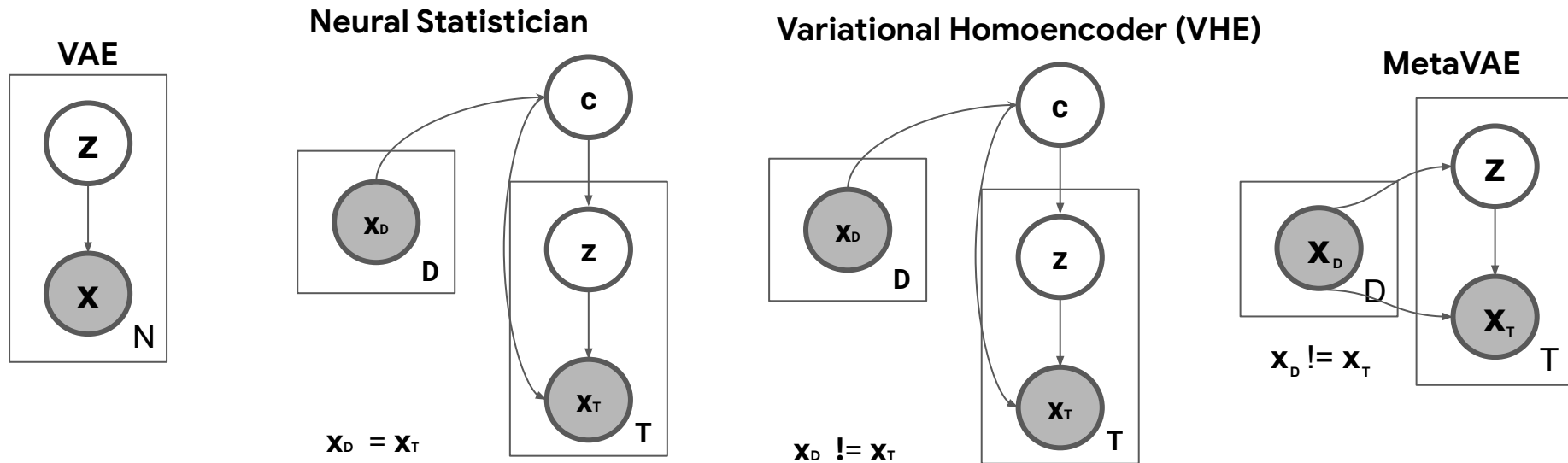
$$\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\} \sim$$



Summary network ingests samples from each dataset

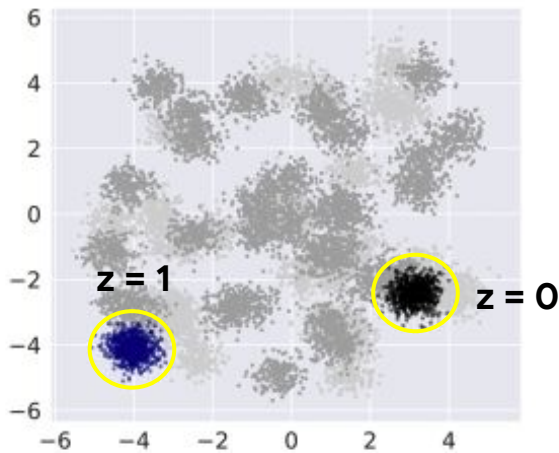
Aggregation network performs inference $\phi = \{\phi_1, \phi_2\}$

Related Work



Avoid restrictive assumption on global prior over datasets $p(c)$

Intuition: Clustering Mixtures of Gaussians

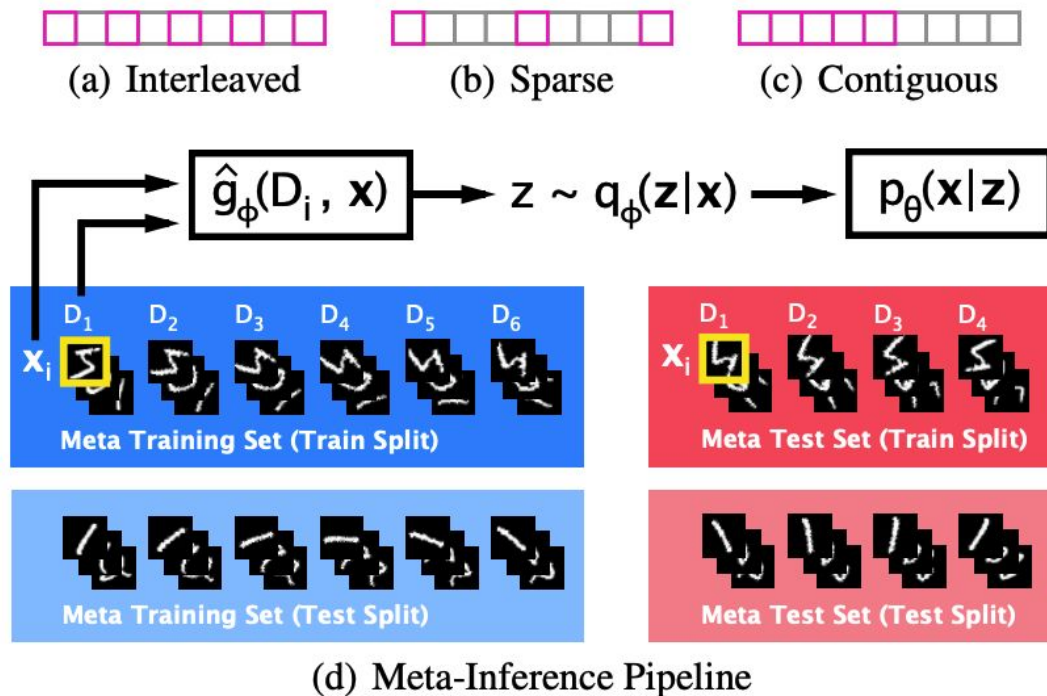


$$\mu_1, \mu_2 \sim \mathcal{U}(-5, 5)$$

$$p_{\mathcal{D}_i}(\mathbf{x}) = \frac{1}{2}\mathcal{N}(\mu_1, 0.1) + \frac{1}{2}\mathcal{N}(\mu_2, 0.1)$$

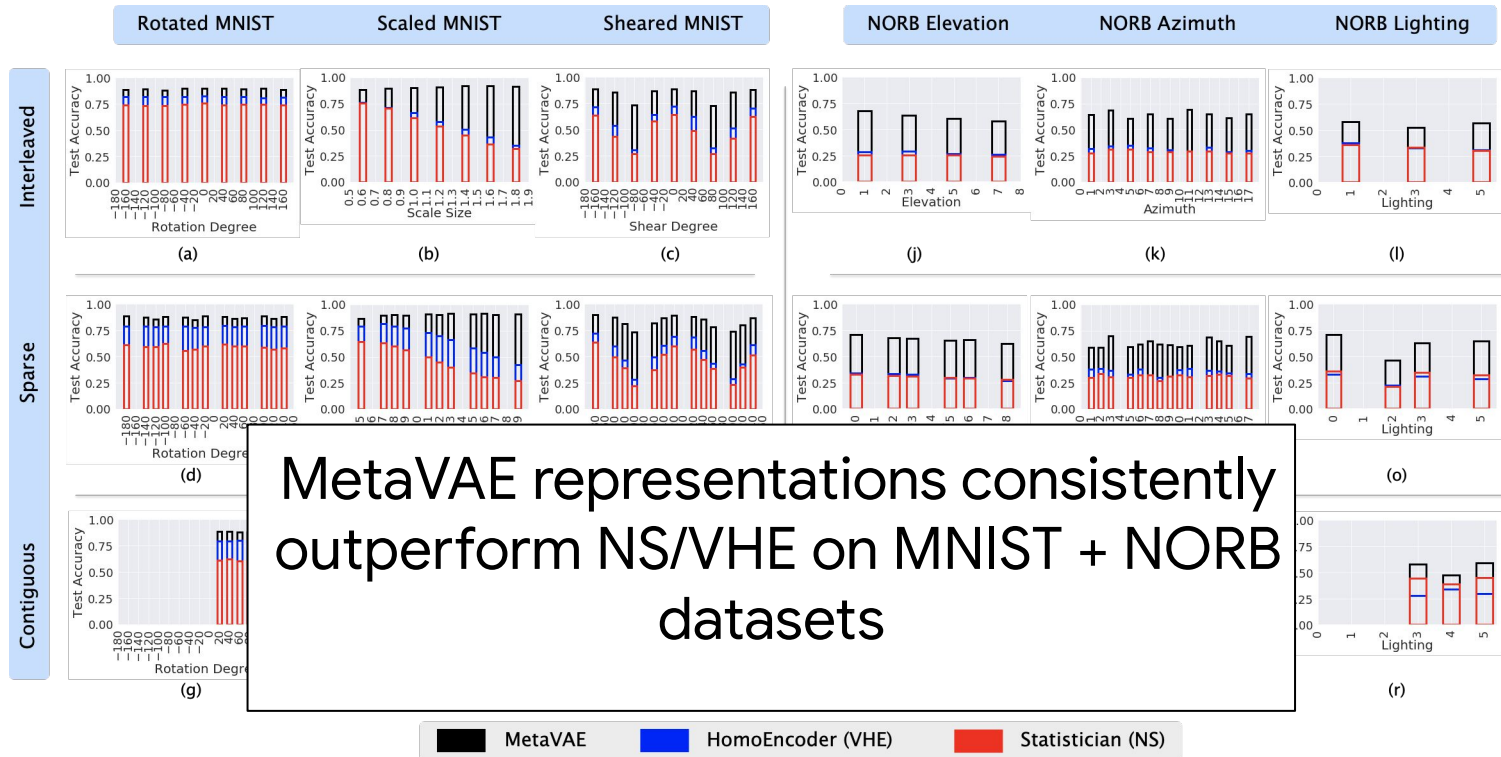
Learns **how to cluster**: for 50 datasets, **MetaVAE** achieves **9.9%** clustering error, while **VAE** gets **27.9%**

Learning Invariant Representations



- Apply various transformations
- Amortize over subsets of transformations, learn representations
- Test representations on held-out transformations (classification)

Invariance Experiment Results



Analysis

Model Dataset	Rotation	Scale	Skew
Rotated MNIST	1.65	4.44	4.09
Scaled MNIST	5.44	2.16	4.92
Skewed MNIST	3.79	4.89	1.47
Model Dataset	Elevation	Azimuth	Lighting
NORB Elevation	0.39	1.16	1.27
NORB Azimuth	1.42	0.44	1.26

MetaVAE representations tend not to change very much within a family of transformations that it was amortized over, as desired.

Conclusion

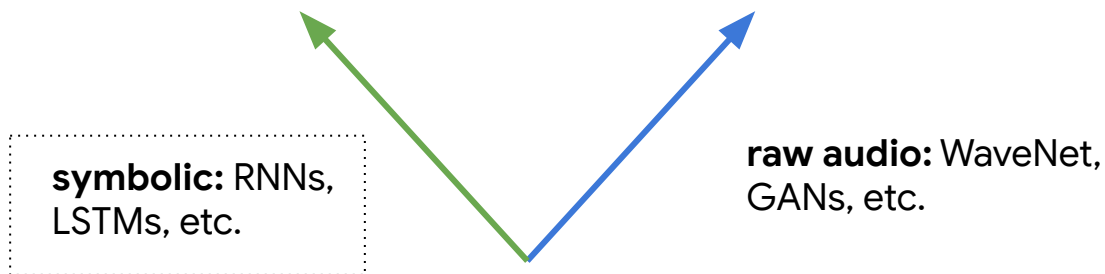
- Limitations
 - No sampling
 - Semi-parametric
 - Arbitrary dataset construction
- Developed an algorithm for a family of probabilistic models: meta-amortized inference paradigm
- MetaVAE learns transferrable representations that generalize well across similar data distributions in downstream tasks
- Paper: <https://arxiv.org/pdf/1902.01950.pdf>



Encoding Musical Style with Transformer Autoencoders

Generative Models for Music

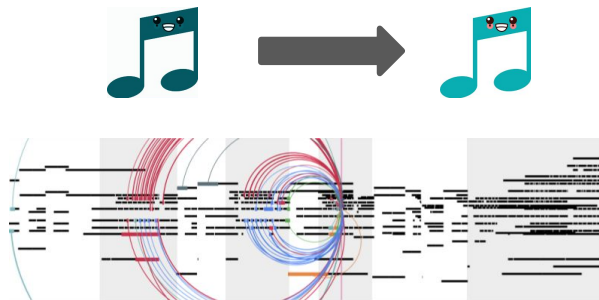
- Generating music is a challenging problem, as music contains structure at multiple timescales.
 - Periodicity, repetition
- Coherence in style and rhythm across (long) time periods!



Music Transformer

- Symbolic: event-based representation that allows for generation of expressive performances (without generating a score)
- Current SOTA in music generation
 - Can generate music over 60 seconds in length
- Attention-based
 - Replaces self-attention with relative attention

What We Want



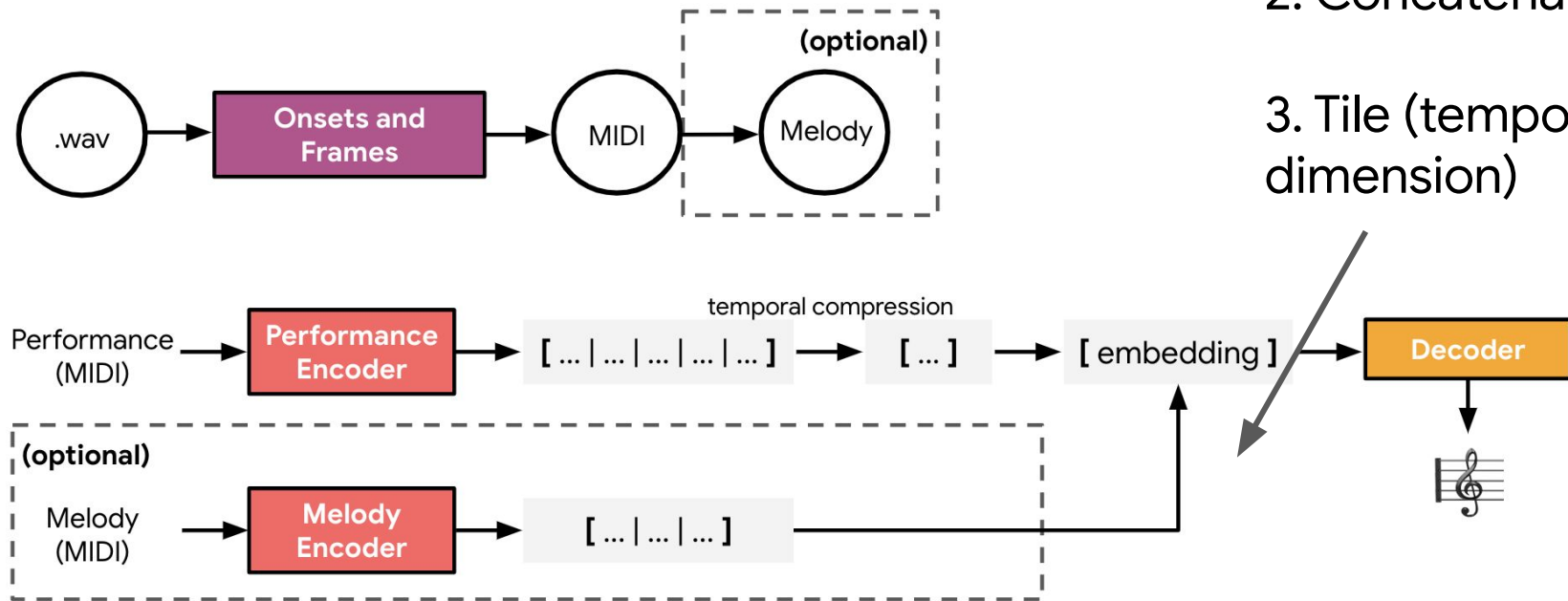
- Control music generation using either (1) performance or (2) melody + perf as conditioning
- Generate pieces that sound similar in style to input pieces!

Transformer Autoencoder

1. Sum

2. Concatenation

3. Tile (temporal dimension)



Quantitative Metrics

MAESTRO	ND	PR	MP	VP	MV	VV	MD	VD	Avg
Melody & perf. (ours)	0.650	0.696	0.634	0.689	0.692	0.732	0.582	0.692	0.67
Perf-only (ours)	0.600	0.695	0.657	0.721	0.664	0.740	0.527	0.648	0.66
Melody-only	0.609	0.693	0.640	0.693	0.582	0.711	0.569	0.636	0.64
Unconditional	0.376	0.461	0.423	0.480	0.384	0.588	0.347	0.520	0.48
Internal Dataset									
Melody & perf (ours)	0.646	0.708	0.610	0.717	0.590	0.706	0.658	0.743	0.67
Perf-only (ours)	0.624	0.646	0.624	0.638	0.422	0.595	0.601	0.702	0.61
Melody-only	0.575	0.707	0.662	0.718	0.583	0.702	0.634	0.707	0.66
Unconditional	0.476	0.580	0.541	0.594	0.400	0.585	0.522	0.623	0.54

Table 4: Average performance across different conditioning. Unconditional performance is described in detail in the study shown in the

Transformer autoencoder (both performance-only and melody & perf) outperform baselines in generating similar pieces!

different conditioning. The metrics are calculated for the listener

Samples



Twinkle, Twinkle melody



Conditioning Performance



Generated Performance:
“Twinkle, Twinkle” in the style
of the above performance



Claire de Lune



Conditioning Performance



Generated Performance:
“Claire de Lune” in the style
of the above performance

Conclusion

- Developed a method for controllable generation with high-level controls for music
 - Demonstrated efficacy both quantitatively and through qualitative listening tests
- Thanks!
 - **Stanford:** Mike Wu, Noah Goodman, Stefano Ermon
 - **Magenta @ Google Brain:** Jesse Engel, Ian Simon, Curtis “Fjord” Hawthorne, Monica Dinculescu

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